

[Time:2.30 Hrs]

[Marks:75]

Please check whether you have got the right question paper.

- N.B: 1. All question are compulsory.
2. Figures to the right indicate full marks.

Q.1 Attempt any four of the following: 20

- A Find the square root of $Z = 3 - 4i$
- B If $Z_1 = 5 - 12i$ and $Z_2 = 8 + 6i$, Find the values of $Z_1 + Z_2$, $Z_1 - Z_2$, $Z_1 * Z_2$, and Z_1 / Z_2 .
- C Find the conjugate, modulus and argument of the following complex numbers: $Z = 8 - 6i$
- D Show that the subset $S = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ of V_3 spans the entire vector space V_3 .
- E Explain Dot product and Cross Product
- F Express $W = (6, -2, 5)$ as a linear combination of $v_1 = (-2, 1, 3)$ and $v_2 = (3, 1, -1)$ and $v_3 = (-1, -2, 1)$.

Q.2 Attempt any four of the following: 20

- A Explain Inner product & outer product
- B Compute the following matrix-vector products $M = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -2 \end{bmatrix}$ and $v = [2, -3, 0]$
- C Compute Matrix Matrix Multiplication : $\begin{bmatrix} 2 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$
- D Let $V = \text{Span} \{[0, 0, 1], [1, 0, 1], [2, 1, 1]\}$. For each of the following vectors, show it belongs to V by writing it as a linear combination of the generators of V . a) $[2, 1, 4]$
- E Verify Rank – Nullity theorem $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x, y) = x + y$
- F For each of the set given below, show the given vectors over \mathbb{R} are linearly dependent. $[1, 2, 0], [2, 4, 1], [0, 0, -1]$

Q.3 Attempt any four of the following: 20

- A Solve the following system of equations by Gaussian elimination method:
 $2x + y + z = 10; 3x + 2y + 3z = 18; x + 4y + 9z = 16$
- B Find $(26, 118)$ and express it in the form $26x + 118y$, where x and $y \in \mathbb{Z}$
- C State Cauchy-Schwartz inequality.
- D Find the vector orthogonal to both $u = (-6, 4, 2)$ and $v = (3, 1, 5)$.
- E Find the characteristic equation and hence Eigen values for $A = \begin{bmatrix} 1 & 3 \\ 4 & 3 \end{bmatrix}$.

F

Find a matrix P which transforms the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to diagonal form

Q.4 Attempt any three of the following:

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- A Express $W = (4, 3)$ as a linear combination of v Vectors $v_1 = (2, 3)$ and $v_2 = (0, 1)$.
- B Solve in $GF(2)$: i.) $1+1+0+1+1$ ii.) $1.1.1+0.1.1+1.1.1+0.0.0$
- C Explain minimum spanning Forest and $Gf(2)$
- D Determine whether the following set of vectors span vector space R^3 , $v_1 = (2, 2, 2)$, $v_2 = (0, 0, 3)$, $v_3 = (0, 1, 1)$
- E Let v_1, v_2, \dots, v_n be mutually orthogonal vectors. Then, $\|v_1 + v_2 + \dots + v_n\|^2 = \|v_1\|^2 + \|v_2\|^2 + \dots + \|v_n\|^2$
- F Let W be a subset of vector space V . Prove that W^\perp is a subspace of R^n .
